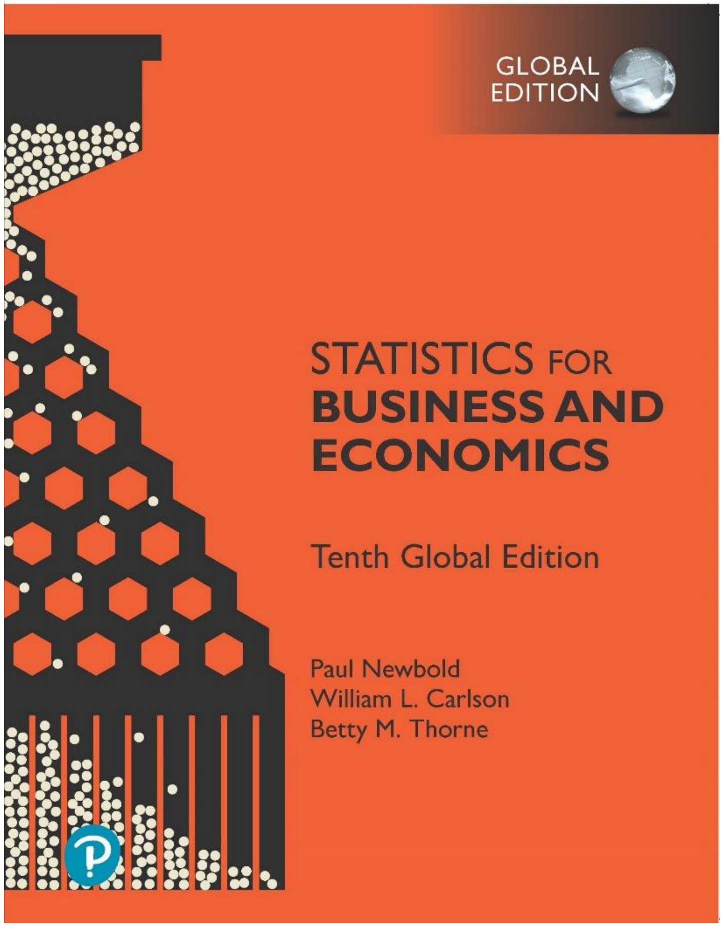
# Statistics for Business and Economics

Tenth Edition, Global Edition




# Chapter 9

## Hypothesis Testing: Single Population

# Chapter Goals

**After completing this chapter, you should be able to:**

- Formulate null and alternative hypotheses for applications involving
  - a single population mean from a normal distribution
  - a single population proportion (large samples)
- Formulate a decision rule for testing a hypothesis
- Know how to use the critical value and *p*-value approaches to test the null hypothesis (for both mean and proportion problems)
- Define Type I and Type II errors

# Section 9.1 Concepts of Hypothesis Testing

- A hypothesis is a claim (assumption) about a population parameter:
  – population mean

    **Example: The mean monthly cell phone bill of this city is $\mu = \$52$**

  – population proportion

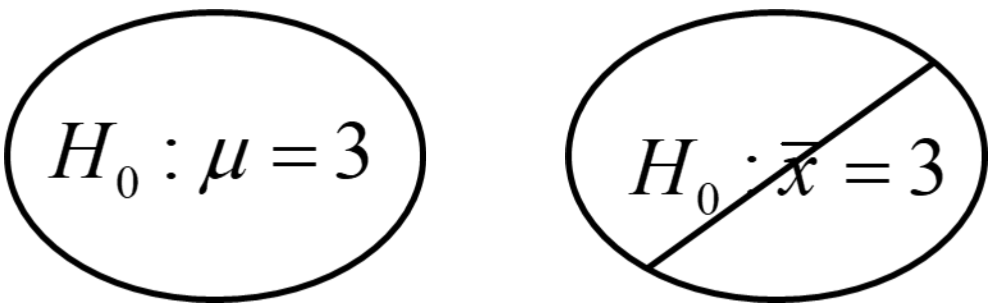
    **Example: The proportion of adults in this city with cell phones is *P* = .88**

# The Null Hypothesis, *H* Sub 0 (1 of 2)

- States the assumption (numerical) to be tested

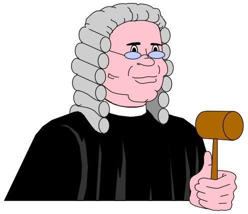
  Example: The average number of TV sets in U.S. Homes is equal to three $(H_0 : \mu = 3)$

- Is always about a population parameter, not about a sample statistic




# The Null Hypothesis, *H* sub 0 (2 of 2)

- Begin with the assumption that the null hypothesis is true
  - Similar to the notion of innocent until proven guilty
- Refers to the status quo
- Always contains "=", "≤" or "≥" sign
- May or may not be rejected



# The Alternative Hypothesis, *H* Sub 1

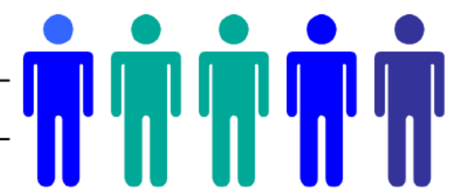
- Is the opposite of the null hypothesis
  - e.g., The average number of TV sets in U.S. homes is not equal to 3 $(H_1 : \mu \neq 3)$
- Challenges the status quo
- Never contains the "$=$", "$\leq$" or "$\geq$" sign
- May or may not be supported
- Is generally the hypothesis that the researcher is trying to support

# Hypothesis Testing Process

Claim: the population mean age is 50. (Null Hypothesis: $H_0: \mu = 50$ )

Population

Now select a random sample



Sample

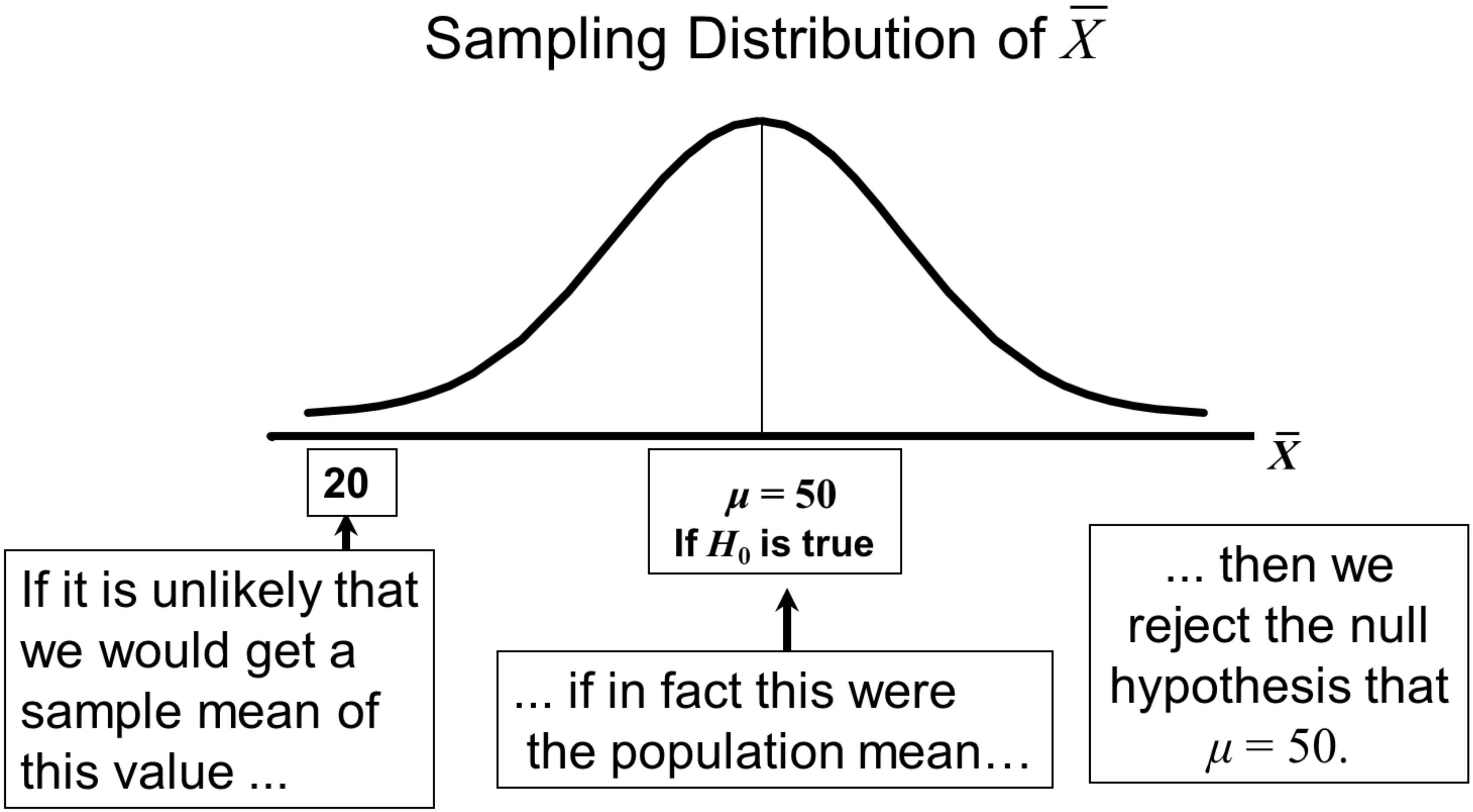
Suppose the sample mean age is 20: $\overline{x} = 20$

Is $\overline{x} = 20$ likely if $\mu = 50$?

If not likely, Reject Null Hypothesis

Pearson

# Reason for Rejecting *H* Sub 0



# Level of Significance, $\alpha$

- **Defines the unlikely values of the sample statistic if the null hypothesis is true**
  - Defines rejection region of the sampling distribution
- Is designated by $\boldsymbol{\alpha}$, (level of significance)
  - Typical values are 0.01, 0.05, or 0.10
- Is selected by the researcher at the beginning
- Provides the critical value(s) of the test

# Level of Significance and the Rejection Region

Level of significance = $\alpha$

$H_0: \mu = 3$

$H_1: \mu \neq 3$

Two-tail test

$H_0: \mu \leq 3$

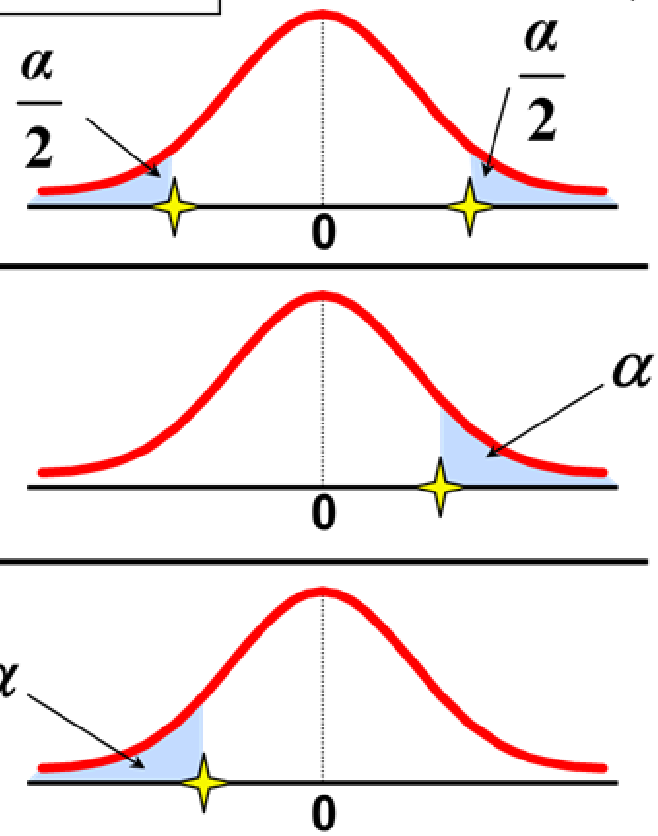
$H_1: \mu > 3$

Upper-tail test

$H_0: \mu \geq 3$

$H_1: \mu < 3$

Lower-tail test




✦ Represents critical value

Rejection region is shaded

# Errors in Making Decisions (1 of 2)

- **Type I Error**
  - Reject a true null hypothesis
  - Considered a serious type of error

The probability of Type I Error is $\boldsymbol{\alpha}$

- Called level of significance of the test
- Set by researcher in advance

# Errors in Making Decisions (2 of 2)

- **Type II Error**
  - Fail to reject a false null hypothesis

The probability of Type II Error is $\beta$

# Outcomes and Probabilities

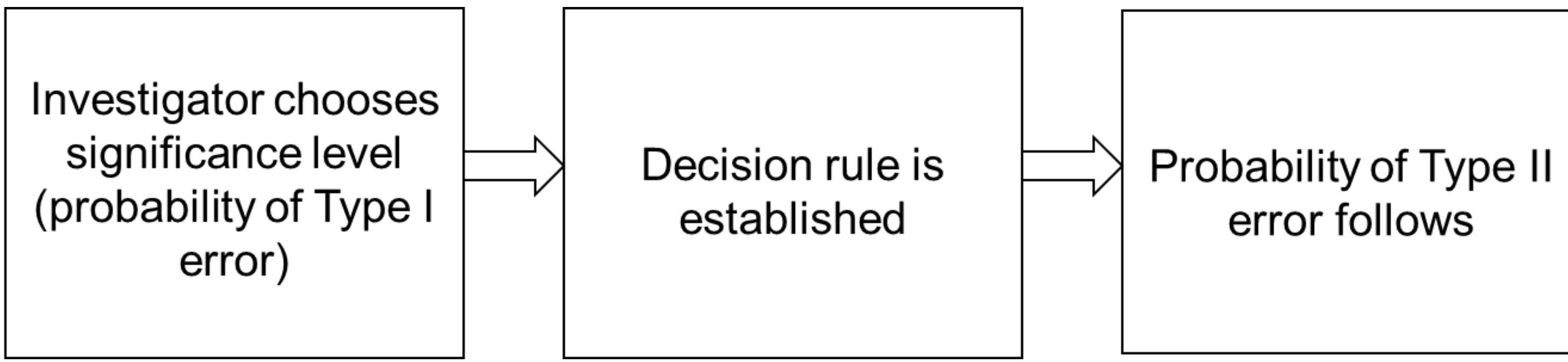
**Key:**
**Outcome**
**(Probability)**

| Possible Hypothesis Test Outcomes | | |
|---|---|---|
| | **Actual Situation** | |
| **Decision** | $H_0$ True | $H_0$ False |
| Fail to Reject $H_0$ | **Correct Decision** $(1 - \alpha)$ | **Type II Error** $(\beta)$ |
| Reject $H_0$ | **Type I Error** $(\alpha)$ | **Correct Decision** $(1 - \beta)$ |

$(1 - \beta)$ is called the power of the test

# Consequences of Fixing the Significance Level of a Test

- Once the significance level $\alpha$ is chosen (generally less than 0.10), the probability of Type II error, $\beta$, can be found.



# Type I & II Error Relationship

- Type I and Type II errors can not happen at the same time
  - Type I error can only occur if $H_0$ is true
  - Type II error can only occur if $H_0$ is false

If Type I error probability $(\alpha) \Uparrow$, then

Type II error probability $(\beta) \Downarrow$

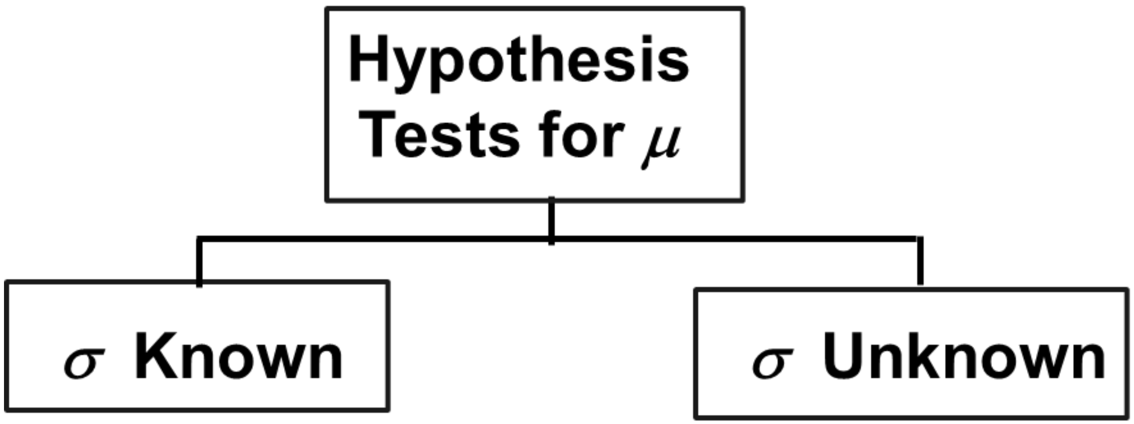
# Factors Affecting Type II Error

- All else equal,
  - $\beta \Uparrow$ when the difference between hypothesized parameter and its true value $\Downarrow$
  - $\beta \Uparrow$ when $\alpha \Downarrow$
  - $\beta \Uparrow$ when $\sigma \Uparrow$
  - $\beta \Uparrow$ when $n \Downarrow$

# Power of the Test

- The power of a test is the probability of rejecting a null hypothesis that is false
- i.e., $\text{Power} = P(\text{Reject } H_0 \mid H_1 \text{ is true})$
  - Power of the test increases as the sample size increases

# Hypothesis Tests for the Mean



# Section 9.2 Tests of the Mean of a Normal Distribution Sigma Known

- Convert sample result $(\bar{x})$ to a $z$ value

**Hypothesis Tests for $\mu$**

**$\sigma$ Known** | **$\sigma$ Unknown**

Consider the test

$$H_0: \mu = \mu_0$$
$$H_1: \mu > \mu_0$$

(Assume the population is normal)

The decision rule is:

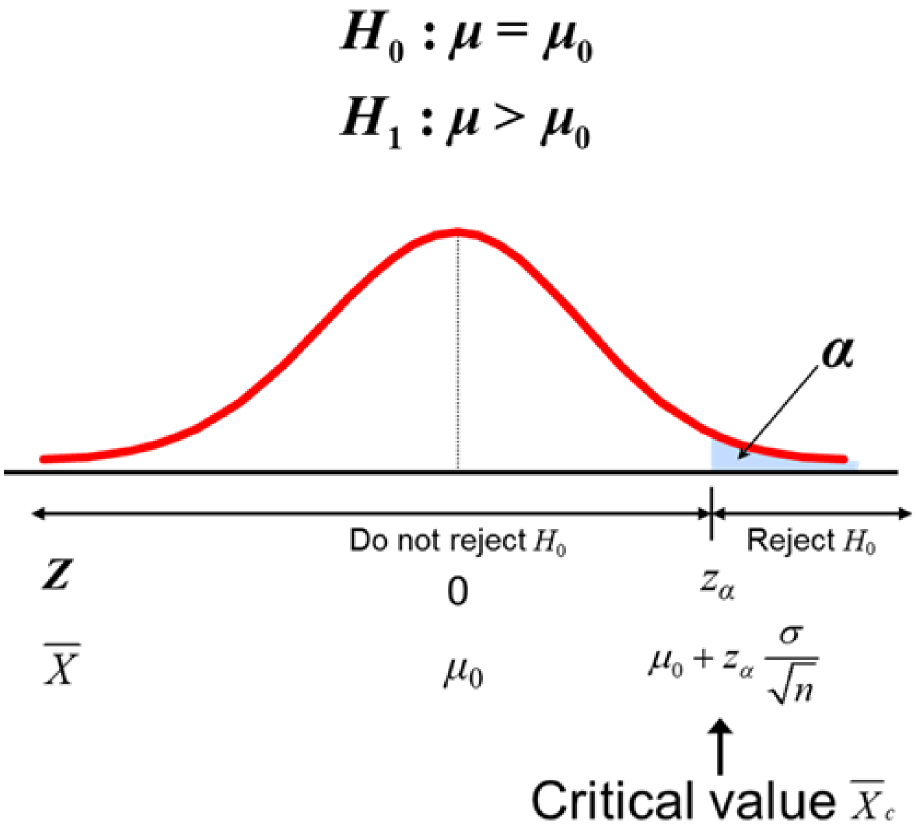
Reject $H_0$ if $z = \dfrac{\bar{x} - \mu_0}{\dfrac{\sigma}{\sqrt{n}}} > z_\alpha$

# Decision Rule

Reject $H_0$ if $z = \dfrac{\bar{x} - \mu_0}{\dfrac{\sigma}{\sqrt{n}}} > z_\alpha$

Alternate rule:

Reject $H_0$ if $\bar{x} > \mu_0 + z_\alpha \dfrac{\sigma}{\sqrt{n}}$



# *p*-Value

- *p*-value: Probability of obtaining a test statistic more extreme ($\leq$ or $\geq$) than the observed sample value given $H_0$ is true
  - Also called observed level of significance
  - Smallest value of ***α*** for which $H_0$ can be rejected

# *p*-Value Approach to Testing

- Convert sample result (e.g., $\bar{x}$) to test statistic (e.g., *z* statistic)
- Obtain the *p*-value
  - For an upper tail test:

$$p\text{-value} = P\left(z > \frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}, \text{ given thet } H_0 \text{ is true}\right)$$

$$= P\left(z > \frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}} \middle| \mu = \mu_0\right)$$

- Decision rule: compare the *p*-value to $\boldsymbol{\alpha}$
  - If *p*-value $< \boldsymbol{\alpha}$, reject $H_0$
  - If *p*-value $\geq \boldsymbol{\alpha}$, do not reject $H_0$

# Example 1: Upper-Tail *Z* Test for Mean Sigma Known

A phone industry manager thinks that customer monthly cell phone bill have increased, and now average over $52 per month. The company wishes to test this claim. (Assume $\sigma = 10$ is known)

Form hypothesis test:

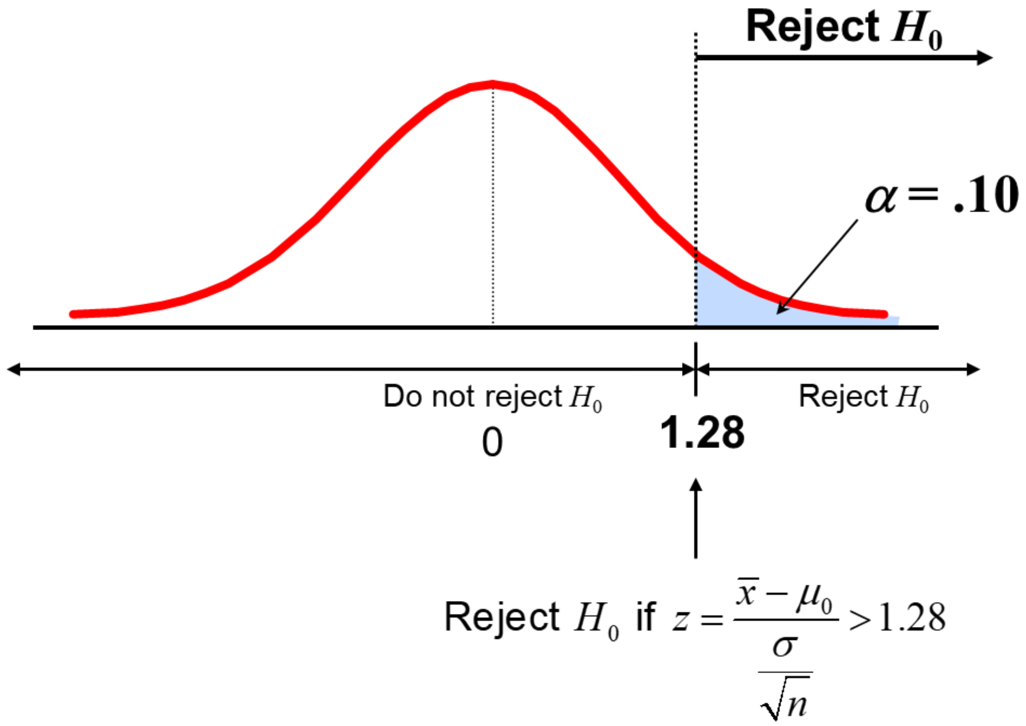
$H_0: \mu \le 52$ the average is not over $52 per month

$H_1: \mu > 52$ the average is greater than $52 per month
(i.e., sufficient evidence exists to support the manager's claim)

# Example 2: Find Rejection Region

- Suppose that $\alpha = .10$ is chosen for this test

Find the rejection region:




# Example 3: Sample Results

Obtain sample and compute the test statistic

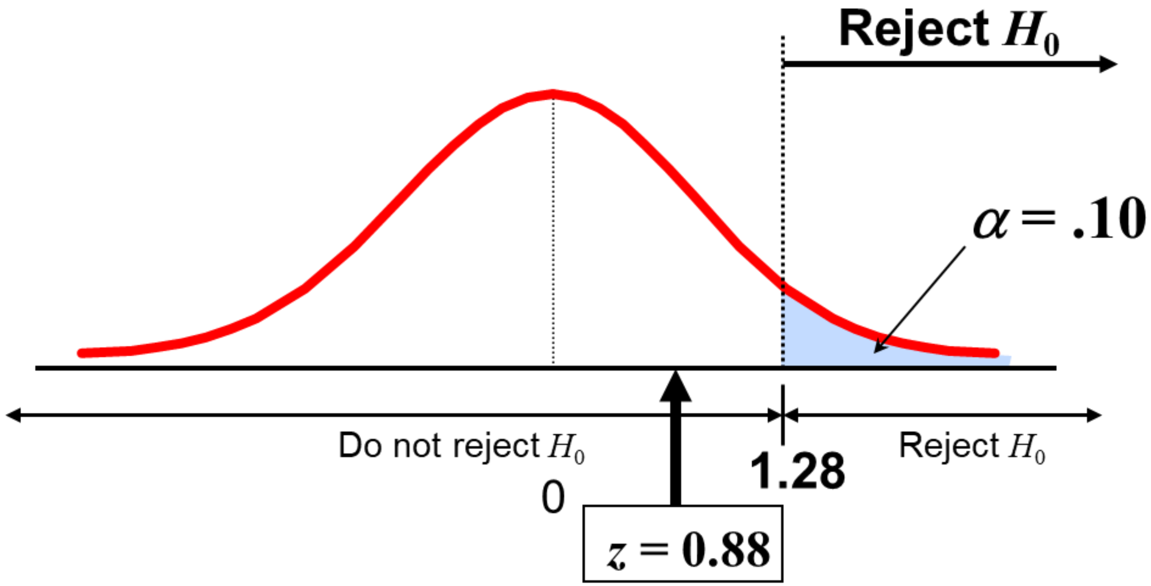
Suppose a sample is taken with the following results: $n = 64,\ \bar{x} = 53.1$ ($\sigma = 10$ was assumed known)

– Using the sample results,

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{53.1 - 52}{\frac{10}{\sqrt{64}}} = 0.88$$

# Example 4: Decision

Reach a decision and interpret the result:



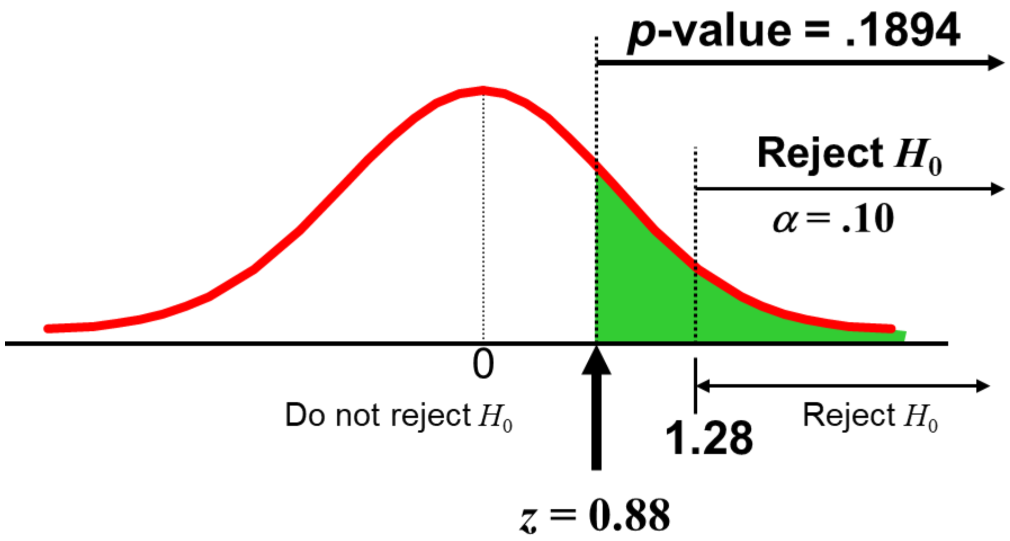

**Do not reject $H_0$ since $z = 0.88 < 1.28$**

i.e.: there is not sufficient evidence that the mean bill is over $52

# Example 5: *p*-Value Solution

Calculate the *p*-value and compare to ***α***

(assuming that $\mu = 52.0$)




$$P(\bar{x} \geq 53.1 \mid \mu = 52.0)$$

$$= P\left( z \geq \frac{53.1 - 52.0}{\frac{10}{\sqrt{64}}} \right)$$

$$= P(z \geq 0.88) = 1 - .8106$$

$$= .1894$$

**Do not reject $H_0$ since *p*-value $= .1894 > \alpha = .10$**

# One-Tail Tests

- In many cases, the alternative hypothesis focuses on one particular direction

$$H_0 : \mu \leq 3$$
$$H_1 : \mu > 3$$

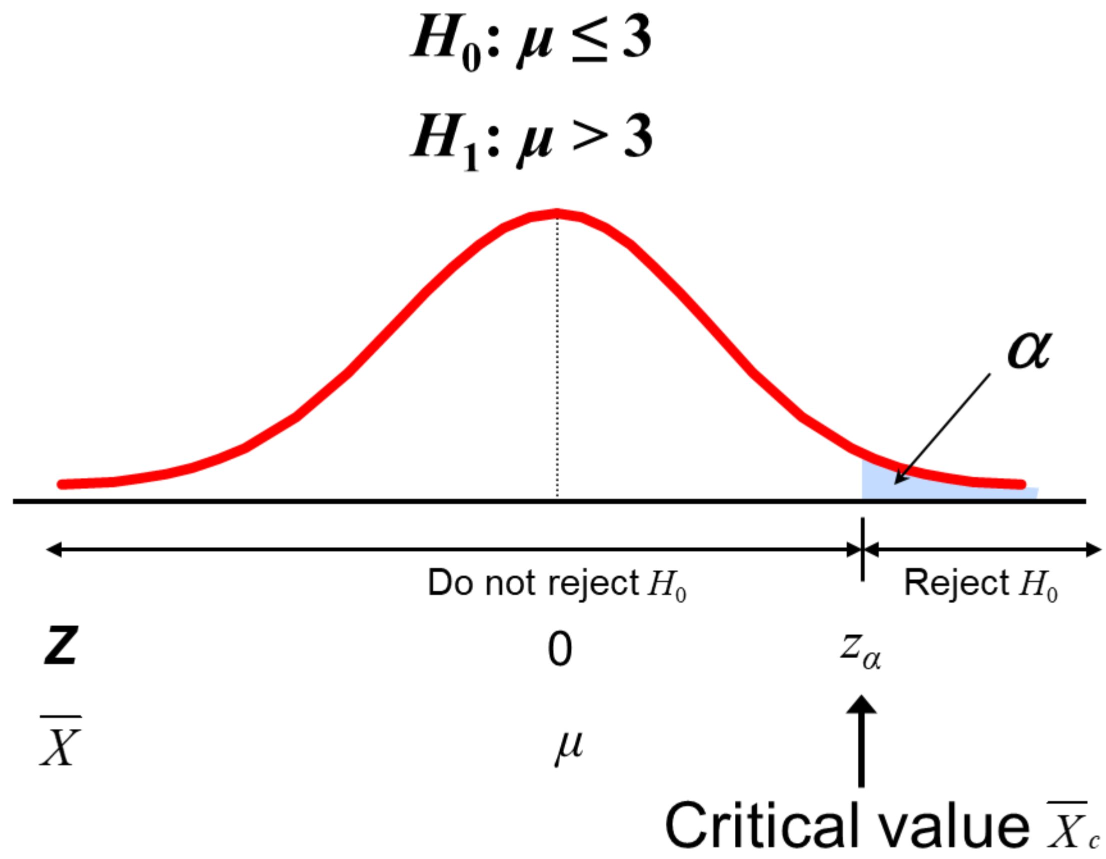
⇒ This is an upper-tail test since the alternative hypothesis is focused on the upper tail above the mean of 3

$$H_0 : \mu \geq 3$$
$$H_1 : \mu < 3$$

⇒ This is a lower-tail test since the alternative hypothesis is focused on the lower tail below the mean of 3

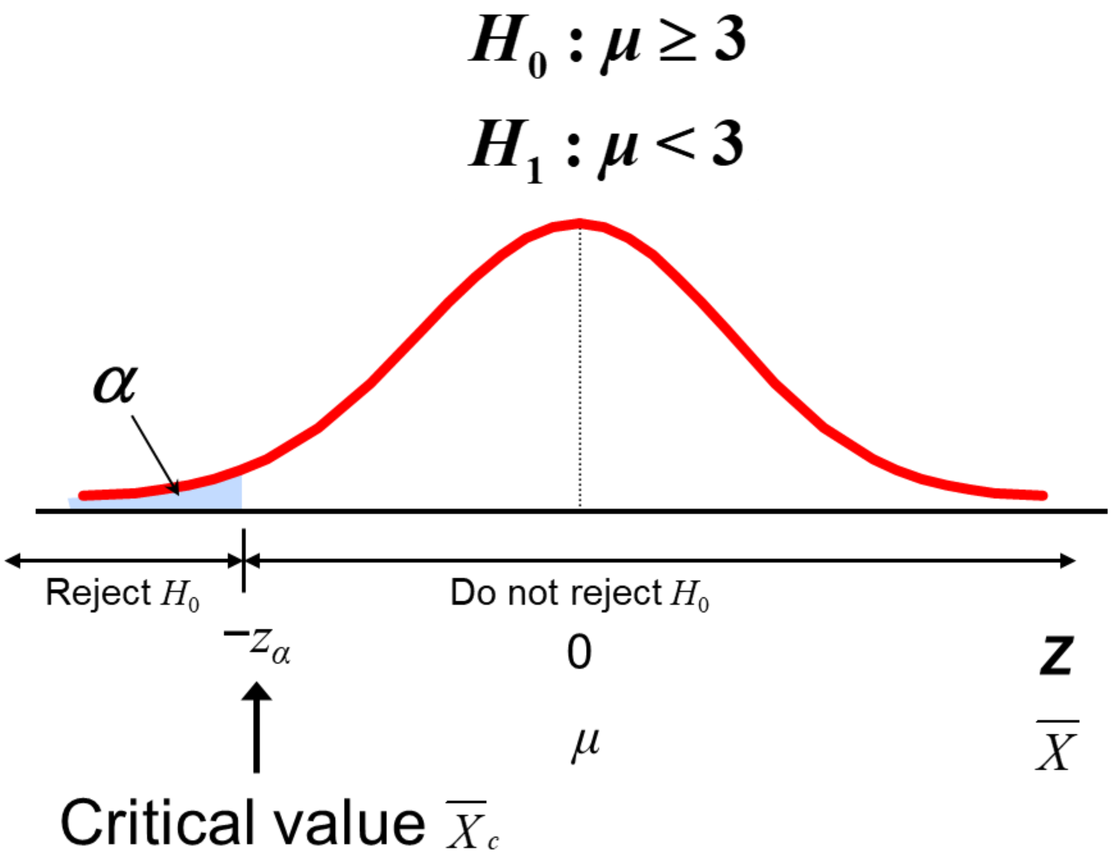
# Upper-Tail Tests

- There is only one critical value, since the rejection area is in only one tail



# Lower-Tail Tests

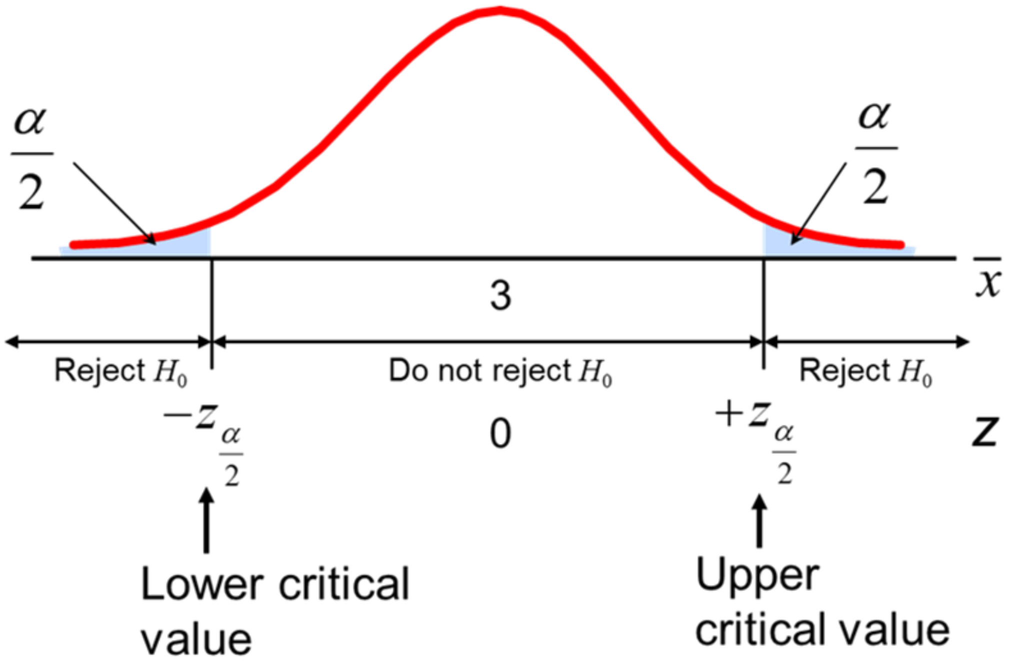
- There is only one critical value, since the rejection area is in only one tail



# Two-Tail Tests

- In some settings, the alternative hypothesis does not specify a unique direction
- There are two critical values, defining the two regions of rejection

$$H_0: \mu = 3$$
$$H_1: \mu \neq 3$$



# Hypothesis Testing Example (1 of 4)

**Test the claim that the true mean # of TV sets in US homes is equal to 3. (Assume $\sigma = 0.8$)**

- State the appropriate null and alternative hypotheses
  - $H_0 : \mu = 3,\ H_1 : \mu \neq 3$ (This is a two tailed test)
- Specify the desired level of significance
  - Suppose that $\alpha = .05$ is chosen for this test
- Choose a sample size
  - Suppose a sample of size $n = 100$ is selected

# Hypothesis Testing Example (2 of 4)

- Determine the appropriate technique
  - $\sigma$ is known so this is a *z* test
- Set up the critical values
  - For $\alpha = .05$ the critical *z* values are $\pm 1.96$
- Collect the data and compute the test statistic
  - Suppose the sample results are
    $n = 100,\ \bar{x} = 2.84$ ($\sigma = 0.8$ is assumed known)

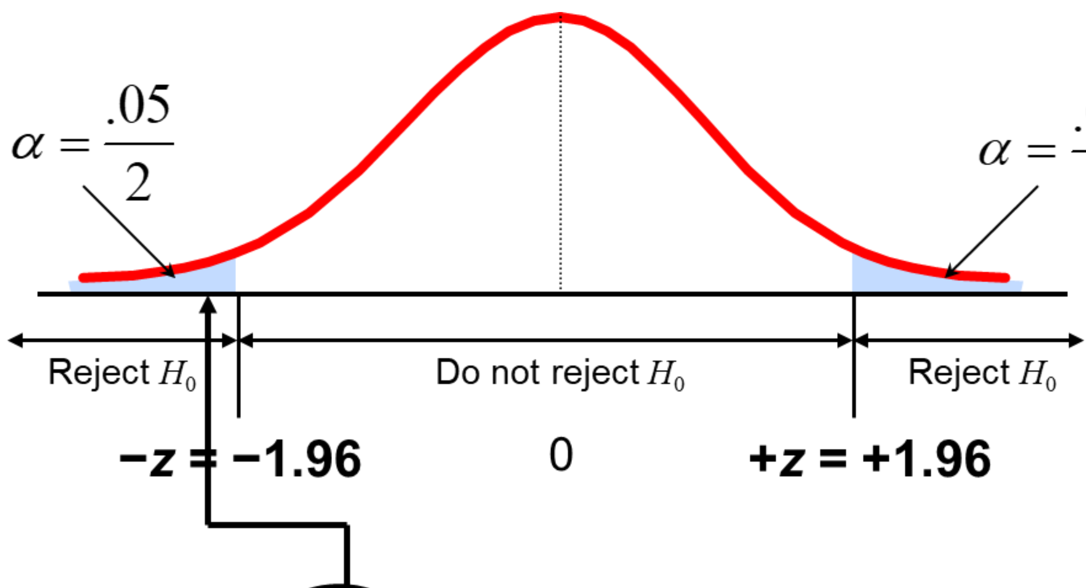
So the test statistic is:

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{.08} = -2.0$$

# Hypothesis Testing Example (3 of 4)

- Is the test statistic in the rejection region?

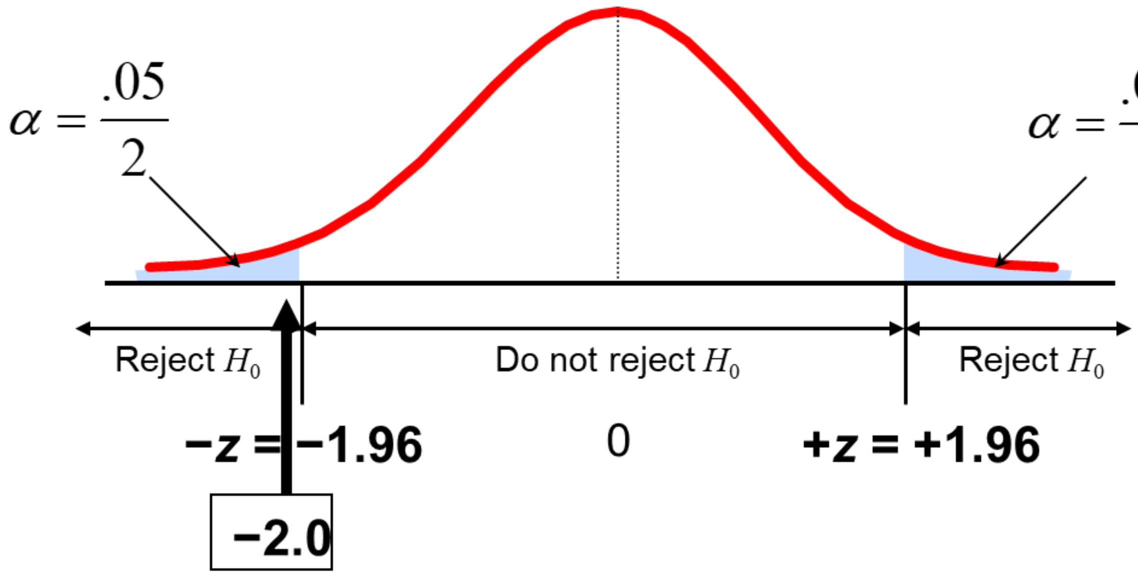
Reject $H_0$ if $z < -1.96$ or $z > 1.96$; otherwise do not reject $H_0$




Here, $z = -2.0 < -1.96$, so the test statistic is in the rejection region

# Hypothesis Testing Example (4 of 4)

- Reach a decision and interpret the result




Since $z = -2.0 < -1.96,$ we reject the null hypothesis and conclude that there is sufficient evidence that the mean number of TVs in US homes is not equal to 3

# Example 6: *p*-Value (1 of 2)

- Example: How likely is it to see a sample mean of 2.84 (or something further from the mean, in either direction) if the true mean is $\mu = 3.0$?

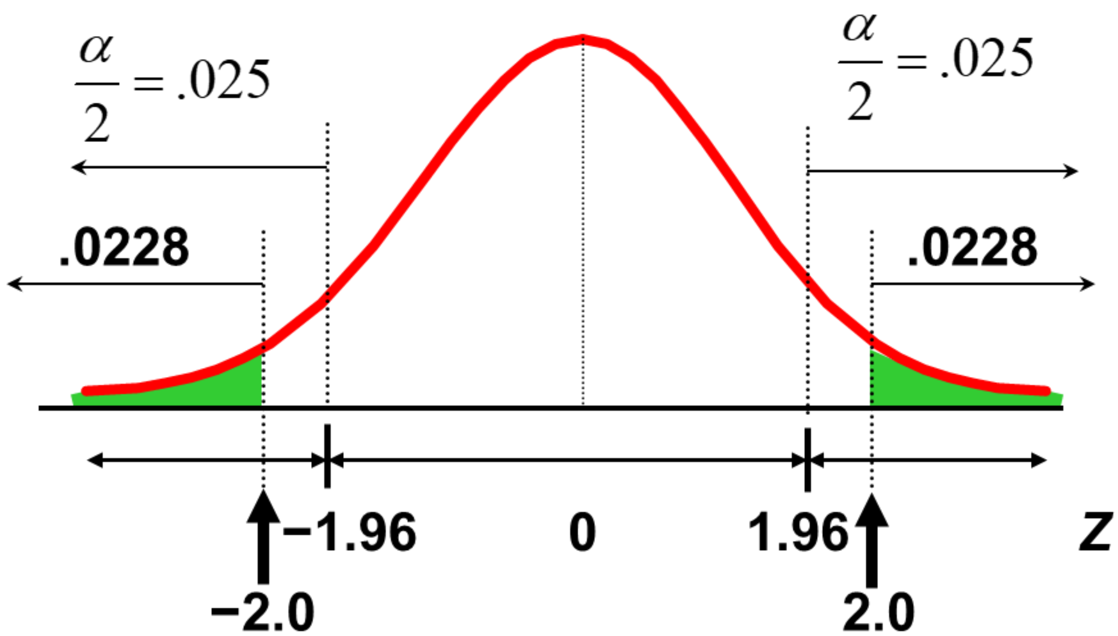
$\bar{x} = 2.84$ is translated to a *z* score of *z* = −2.0

$$P(z < -2.0) = .0228$$

$$P(z > 2.0) = .0228$$

***p*-value**

$$= \mathbf{.0228 + .0228 = .0456}$$



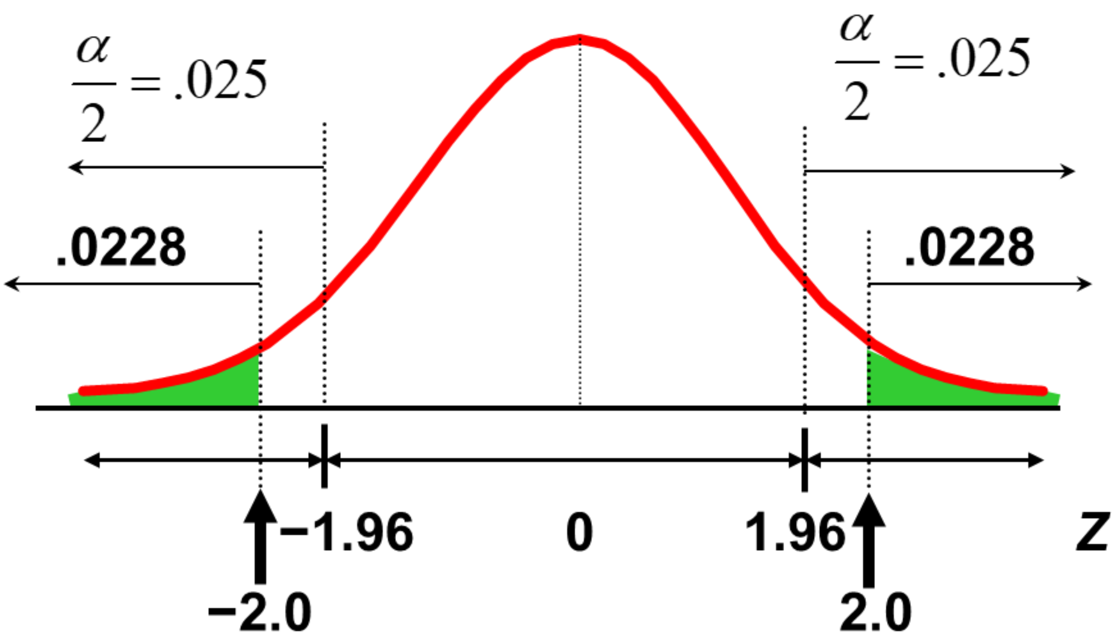
# Example 6: *p*-Value (2 of 2)

- Compare the *p*-value to $\boldsymbol{\alpha}$
  - If $p\text{-value} < \boldsymbol{\alpha}$, reject $H_0$
  - If $p\text{-value} \geq \boldsymbol{\alpha}$, do not reject $H_0$

Here: $p\text{-value} = .0456$

$$\boldsymbol{\alpha} = .05$$

**Since .0456 < .05, we reject the null hypothesis**



# Section 9.3 Tests of the Mean of a Normal Population Sigma Unknown (1 of 2)

- Convert sample result $(\bar{x})$ to a *t* test statistic

**Hypothesis Tests for $\mu$**

**$\sigma$ Known**

**$\sigma$ Unknown**

Consider the test

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu > \mu_0$$

(Assume the population is normal)

The decision rule is:

Reject $H_0$ if $t = \dfrac{\bar{x} - \mu_0}{\dfrac{s}{\sqrt{n}}} > t_{n-1,\, \alpha}$

# Section 9.3 Tests of the Mean of a Normal Population sigma unknown (2 of 2)

- For a two-tailed test:

Consider the test

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu \neq \mu_0$$

(Assume the population is normal, and the population variance is unknown)

The decision rule is:

Reject $H_0$ if $t = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} < -t_{n-1,\frac{\alpha}{2}}$ or if $t = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} > t_{n-1,\frac{\alpha}{2}}$

# Example 7: Two-Tail Test Sigma Unknown

The average cost of a hotel room in Chicago is said to be \$168 per night. A random sample of 25 hotels resulted in $\bar{x} = \$172.50$ and $s = \$15.40$. Test at the $\alpha = 0.05$ level.

(Assume the population distribution is normal)

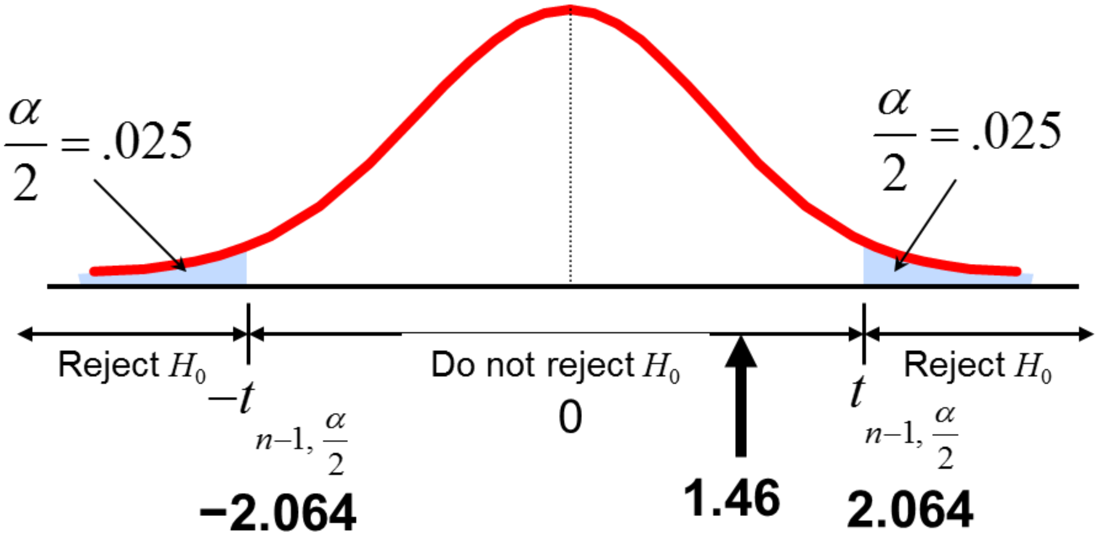
$$H_0: \mu = 168$$

$$H_1: \mu \neq 168$$

# Example Solution: Two-Tail Test

$H_0 : \mu = 168$

$H_1 : \mu \neq 168$




- $\alpha = 0.05$
- $n = 25$
- $\sigma$ is unknown, so use a $t$ statistic →

$$t_{n-1} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

- Critical Value:

$t_{24,.025} = \pm 2.064$

**Do not reject $H_0$ :** not sufficient evidence that true mean cost is different than $168

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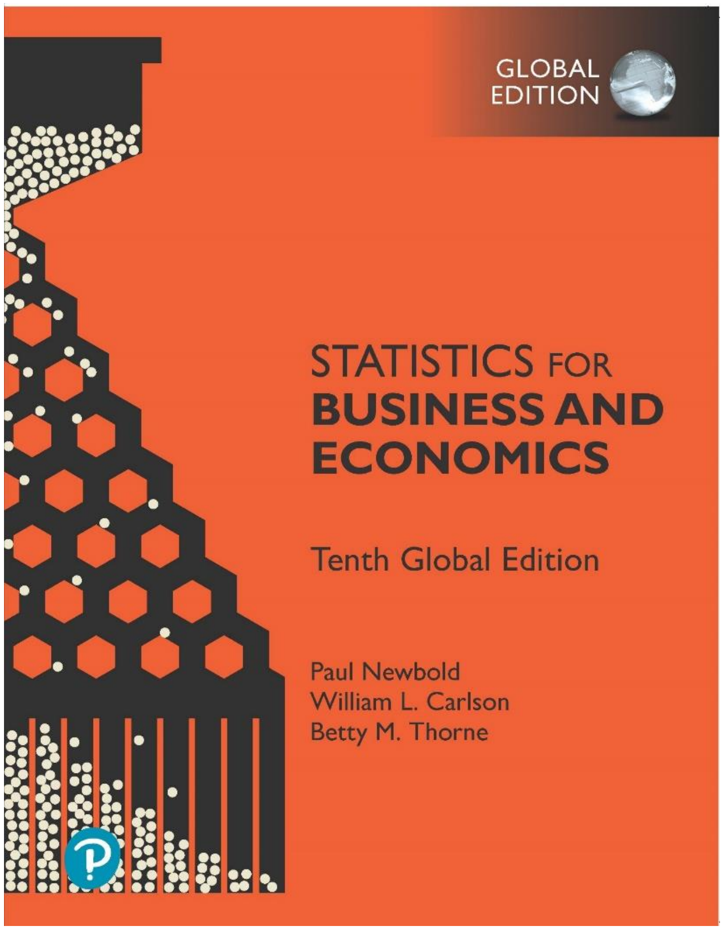



## Chapter 15

Analysis of Variance

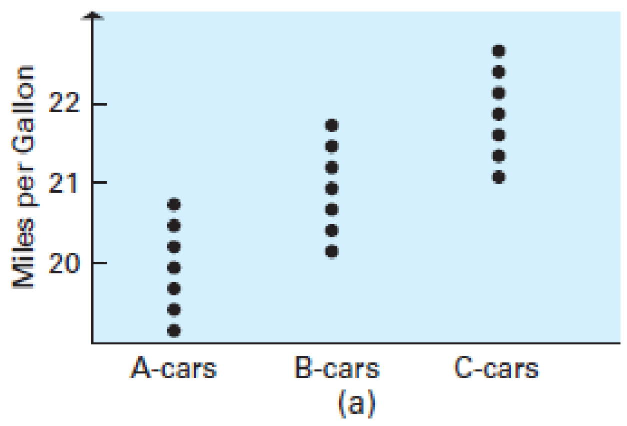
# Chapter Goals

- Recognize situations in which to use analysis of variance
- Understand different analysis of variance designs
- Perform a one-way analysis of variance and interpret the results

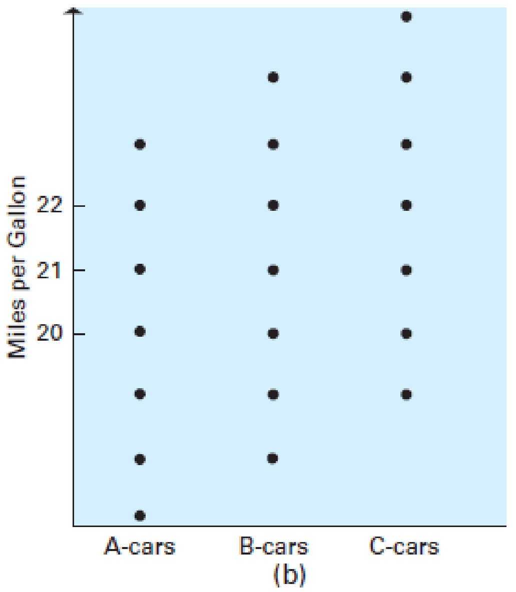
# Section 15.1 Comparison of Several Population Means (1 of 2)

- Tests were presented in Chapter 10 for the difference between two population means
- In this chapter these procedures are extended to tests for the equality of more than two population means
- The null hypothesis is that the population means are all the same
- The critical factor is the variability involved in the data
  - If the variability around the sample means is small compared with the variability among the sample means, we reject the null hypothesis

# Section 15.1 Comparison of Several Population Means (2 of 2)




- Small variation around the sample means compared to the variation among the sample means




- Large variation around the sample means compared to the variation among the sample means

# Section 15.2 One-Way Analysis of Variance

- Evaluate the difference among the means of three or more groups

  Examples: Average production for 1st, 2nd, and 3rd shifts
  Expected mileage for five brands of tires

- Assumptions
  - Populations are normally distributed
  - Populations have equal variances
  - Samples are randomly and independently drawn

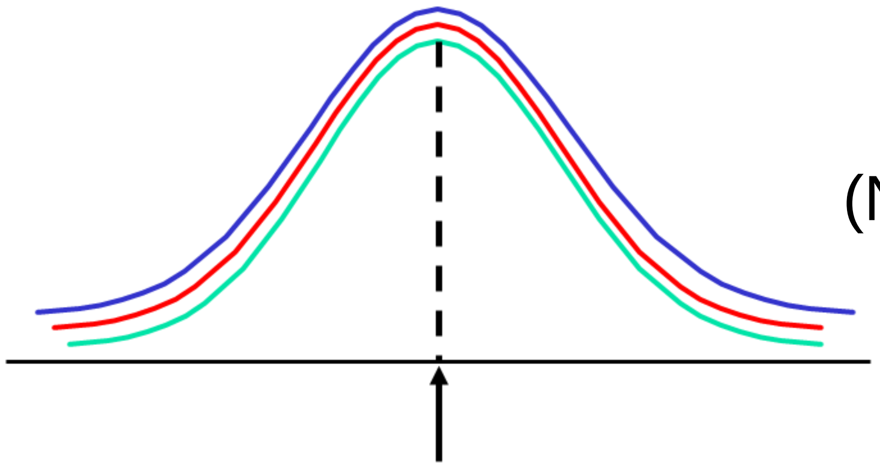
# Hypotheses of One-Way ANOVA

- $H_0 : \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_k$
  - All population means are equal
  - i.e., no variation in means between groups
- $H_1 : \mu_i \neq \mu_j$ for at least one *i* , *j* pair.
  - At least one population mean is different
  - i.e., there is variation between groups
  - Does not mean that all population means are different (some pairs may be the same)

# One-Way ANOVA (1 of 2)

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_k$$

$H_1$ : Not all $\mu_i$ are the same



$$\mu_1 = \mu_2 = \mu_3$$

All Means are the same:
The Null Hypothesis is True
(No variation between groups)

# One-Way ANOVA (2 of 2)

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_K$$

$H_1$ : Not all $\mu_i$ are the same.

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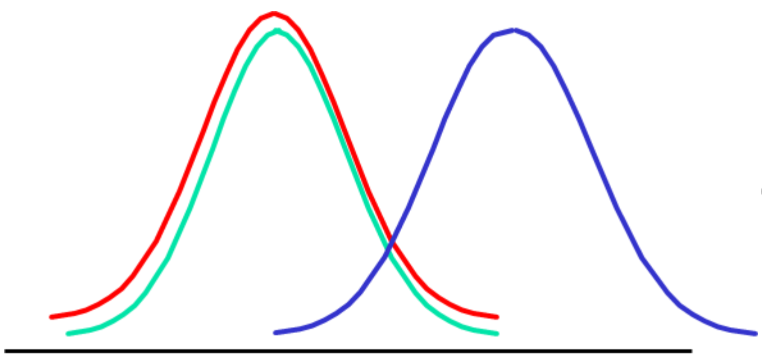
At least one mean is different:

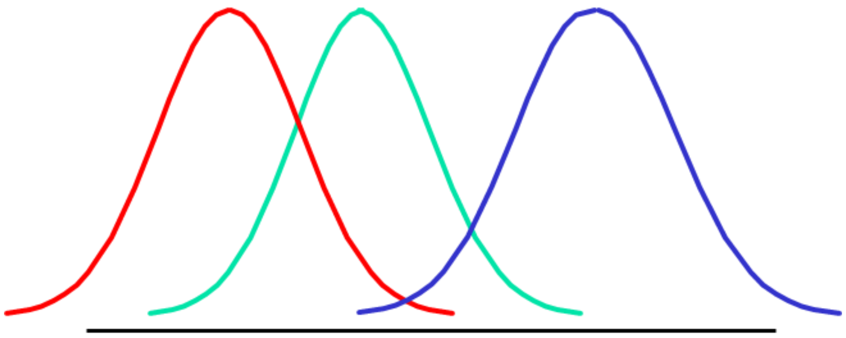
The Null Hypothesis is Not true

(Variation is present between groups)



$$\mu_1 = \mu_2 \neq \mu_3$$

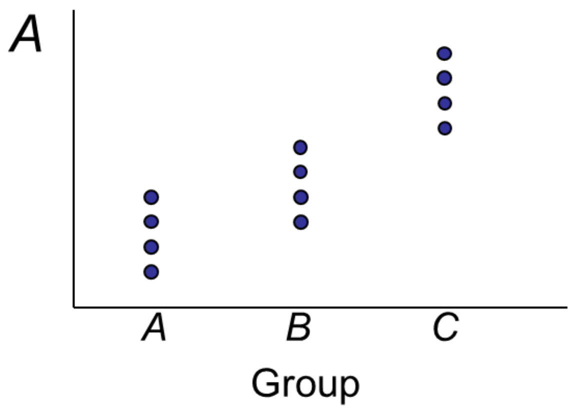
or



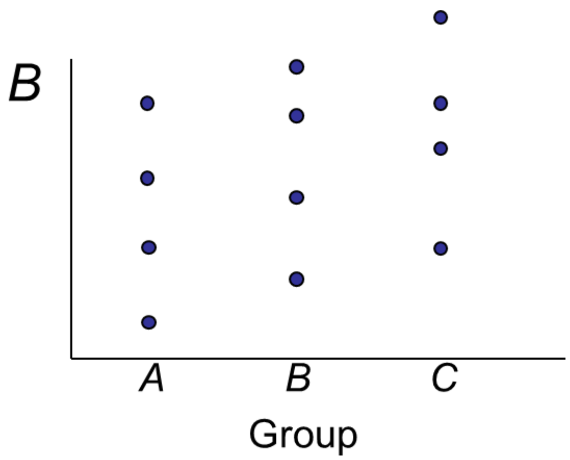
$$\mu_1 \neq \mu_2 \neq \mu_3$$

# Variability

- The variability of the data is key factor to test the equality of means
- In each case below, the means may look different, but a large variation within groups in *B* makes the evidence that the means are different weak




Small variation within groups




Large variation within groups

# Sum of Squares Decomposition (1 of 2)

- Total variation can be split into two parts:

$$\text{SST} = \text{SSW} + \text{SSG}$$

SST = Total Sum of Squares

Total Variation = the aggregate dispersion of the individual data values across the various groups

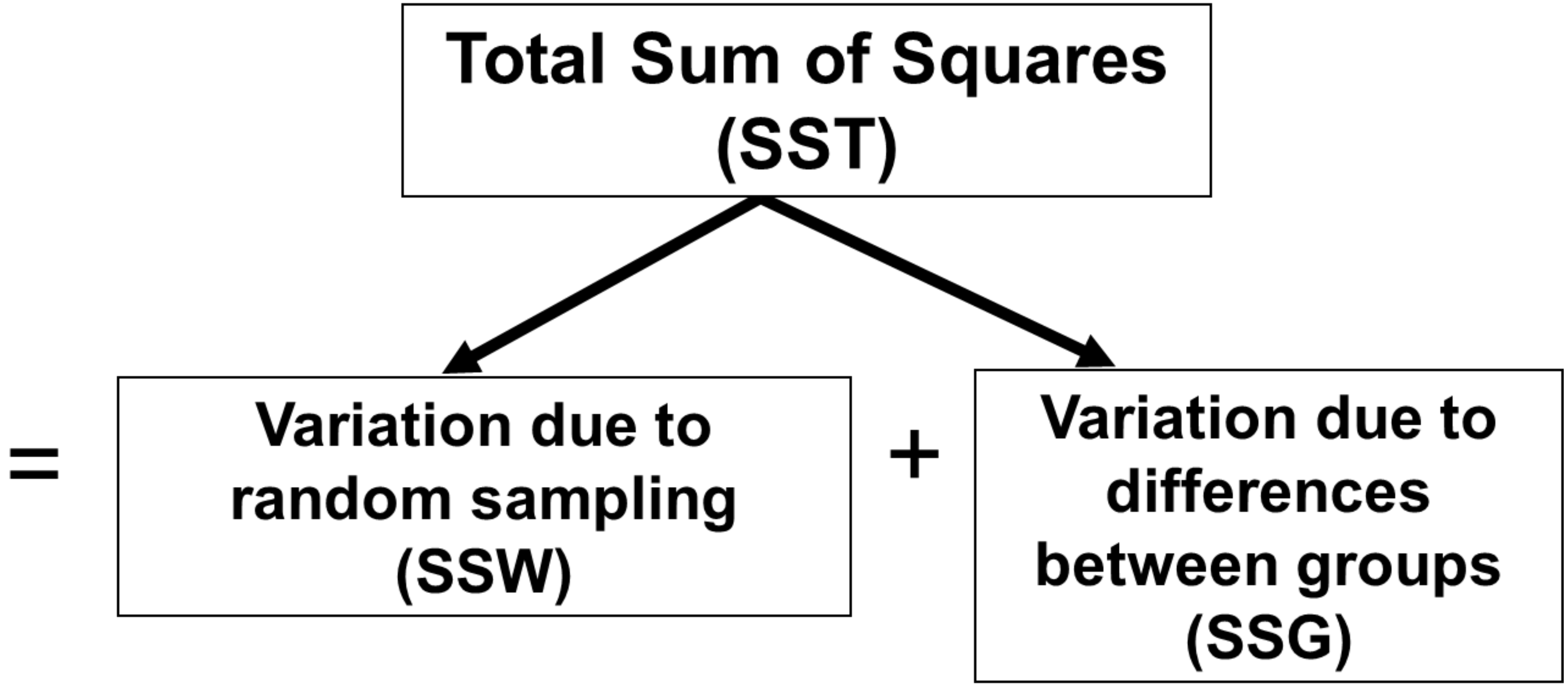
SSW = Sum of Squares Within Groups

Within-Group Variation = dispersion that exists among the data values within a particular group

SSG = Sum of Squares Between Groups

Between-Group Variation = dispersion between the group sample means

# Sum of Squares Decomposition (2 of 2)



# Total Sum of Squares (1 of 2)

$$\text{SST} = \text{SSW} + \text{SSG}$$

$$\text{SST} = \sum_{i=1}^{K}\sum_{j=1}^{n_i}\left(x_{ij} - \overline{\overline{x}}\right)^2$$

Where:

SST = Total sum of squares

$K$ = number of groups (levels or treatments)

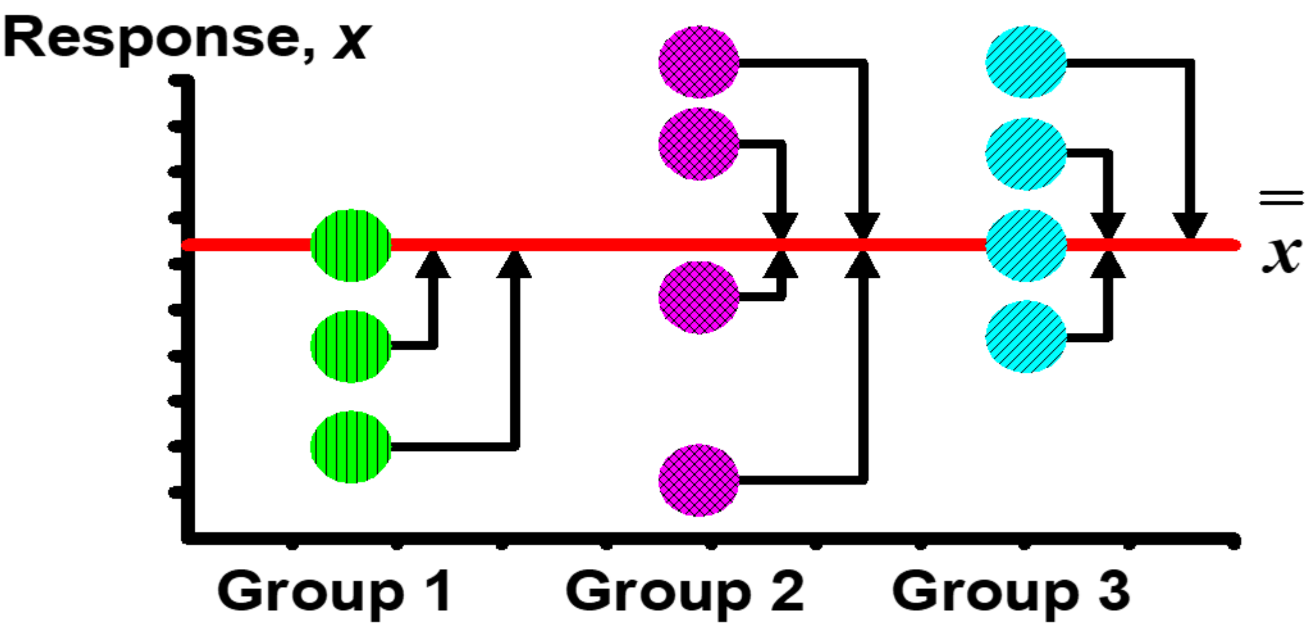
$n_i$ = number of observations in group $i$

$x_{ij} = j^{th}$ observation from group $i$

$\overline{\overline{x}}$ = overall sample mean

# Total Sum of Squares (2 of 2)

$$SST = \left(x_{11} - \bar{\bar{x}}\right)^2 + \left(x_{12} - \bar{\bar{x}}\right)^2 + \cdots + \left(x_{Kn_K} - \bar{\bar{x}}\right)^2$$



# Within-Group Variation (1 of 3)

$$\text{SST} = \text{SSW} + \text{SSG}$$

$$\text{SSW} = \sum_{i=1}^{K} \sum_{j=1}^{n_i} \left( x_{ij} - \bar{x}_i \right)^2$$

Where:

SSW = Sum of squares within groups

$K$ = number of groups

$n_i$ = sample size from group *i*

$\bar{x}_i$ = sample mean from group *i*

$x_{ij} = j^{\text{th}}$ observation in group *i*

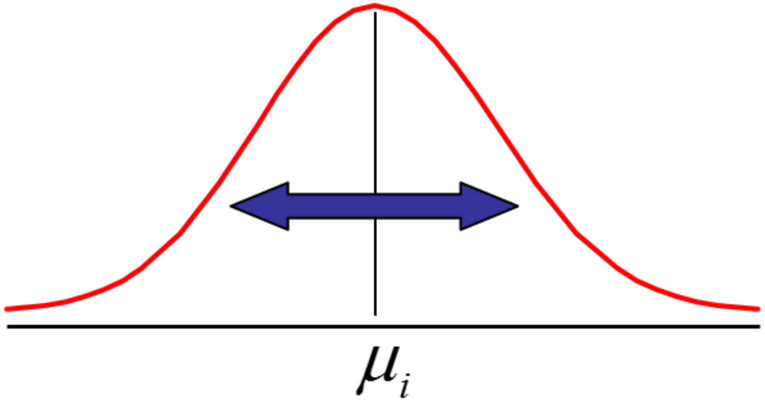
# Within-Group Variation (2 of 3)

$$\text{SSW} = \sum_{i=1}^{K} \sum_{j=1}^{n_i} \left( x_{ij} - \bar{x}_i \right)^2$$

Summing the variation within each group and then adding over all groups

$$\text{MSW} = \frac{\text{SSW}}{n - K}$$

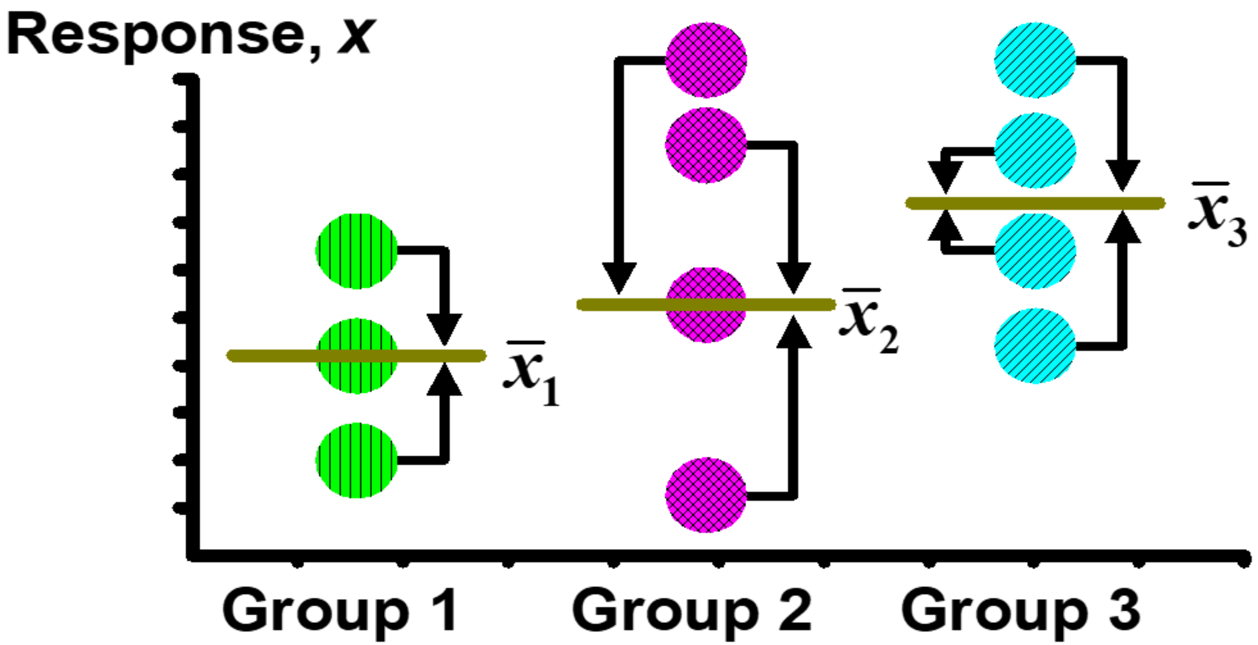
$$\text{Mean Square Within} = \frac{\text{SSW}}{\text{degrees of freedom}}$$




Pearson

# Within-Group Variation (3 of 3)

$$\text{SSW} = \left(x_{11} - \bar{x}_1\right)^2 + \left(x_{12} - \bar{x}_1\right)^2 + \ldots + \left(x_{Kn_K} - \bar{x}_K\right)^2$$



# Between-Group Variation (1 of 3)

$$\text{SST} = \text{SSW} + \text{SSG}$$

$$\text{SSG} = \sum_{i=1}^{K} n_i \left( \bar{x}_i - \bar{\bar{x}} \right)^2$$

Where:

SSG = Sum of squares between groups

$K$ = number of groups

$n_i$ = sample size from group *i*

$\bar{x}_i$ = sample mean from group *i*

$\bar{\bar{x}}$ = grand mean (mean of all data values)

# Between-Group Variation (2 of 3)

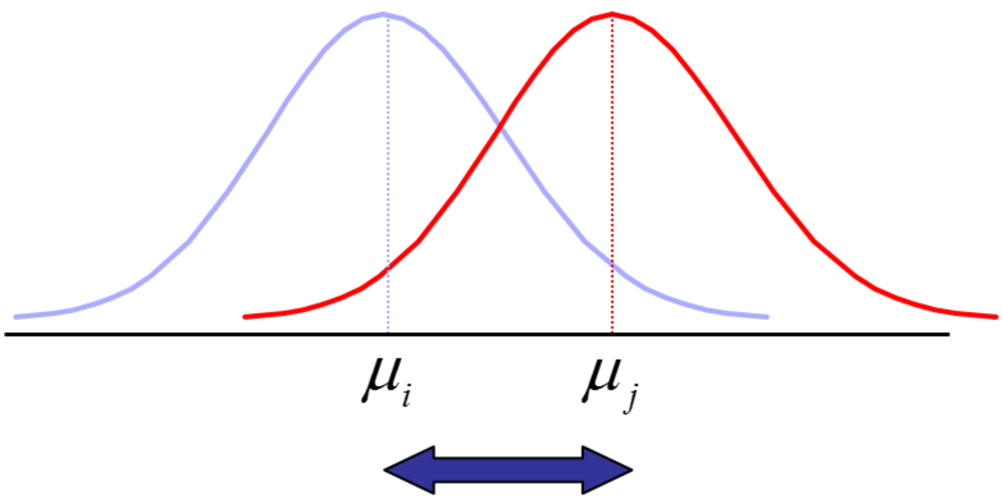
$$\text{SSG} = \sum_{i=1}^{K} n_i \left( \bar{x}_i - \bar{\bar{x}} \right)^2$$

Variation Due to Differences Between Groups

$$\text{MSG} = \frac{\text{SSG}}{K-1}$$

Mean Square Between Groups

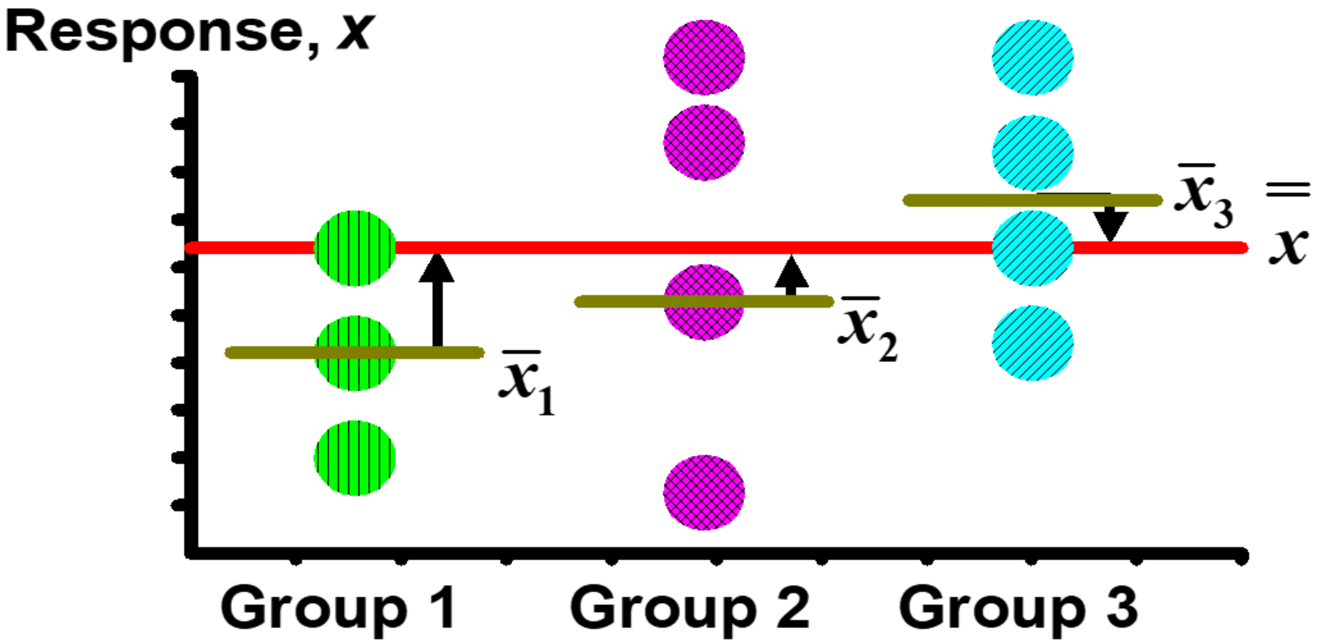
$$= \frac{\text{SSG}}{\text{degrees of freedom}}$$




Pearson

# Between-Group Variation (3 of 3)

$$\text{SSG} = n_1\left(\bar{x}_1 - \bar{\bar{x}}\right)^2 + n_2\left(\bar{x}_2 - \bar{\bar{x}}\right)^2 + \ldots + n_K\left(\bar{x}_K - \bar{\bar{x}}\right)^2$$



# Obtaining the Mean Squares

$$\text{MST} = \frac{\text{SST}}{n-1}$$

$$\text{MSW} = \frac{\text{SSW}}{n-K}$$

$$\text{MSG} = \frac{\text{SSG}}{K-1}$$

Where $n$ = sum of the sample sizes from all groups

$K$ = number of populations

# One-Way ANOVA Table

| Source of Variation | SS | df | MS (Variance) | *F* ratio |
|---|---|---|---|---|
| Between Groups | SSG | $K-1$ | $MSG = \frac{SSG}{K-1}$ | $F = \frac{MSG}{MSW}$ |
| Within Groups | SSW | $n-K$ | $MSW = \frac{SSW}{n-K}$ | |
| Total | SST = SSG + SSW | $n-1$ | | |

$K$ = number of groups

$n$ = sum of the sample sizes from all groups

df = degrees of freedom

# One-Factor ANOVA *F* Test Statistic

$$H_0 : \mu_1 = \mu_2 = \ldots = \mu_K$$

$H_1 :$ At least two population means are different

- Test statistic

$$F = \frac{\text{MSG}}{\text{MSW}}$$

MSG is mean squares between variances

MSW is mean squares within variances

- Degrees of freedom
  - $\text{df}_1 = K - 1$ (*K* = number of groups)
  - $\text{df}_2 = n - K$ (*n* = sum of sample sizes from all groups)

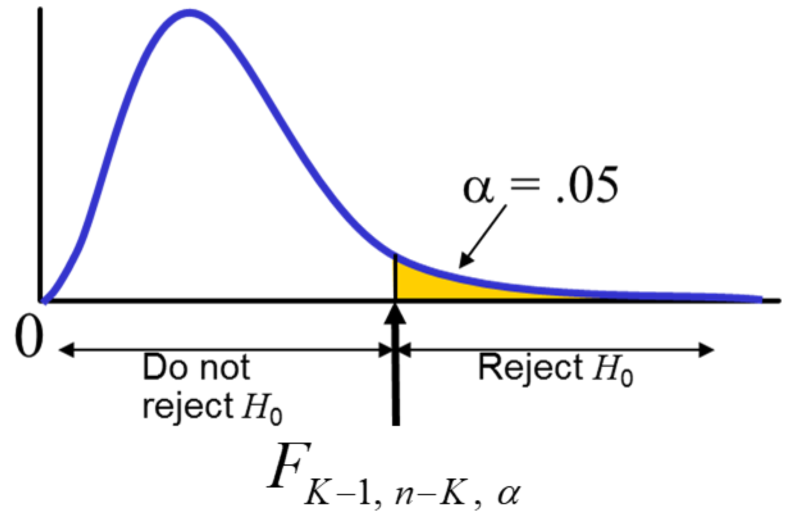
# Interpreting the *F* Statistic

- The *F* statistic is the ratio of the between estimate of variance and the within estimate of variance
  - The ratio must always be positive
  - $\text{df}_1 = K - 1$ will typically be small
  - $\text{df}_2 = n - K$ will typically be large

Decision Rule:

- Reject $H_0$ if

$$F > F_{K-1,\, n-K,\, \alpha}$$



# One-Factor ANOVA *F* Test Example

You want to see if three different golf clubs yield different distances. You randomly select five measurements from trials on an automated driving machine for each club. At the .05 significance level, is there a difference in mean distance?

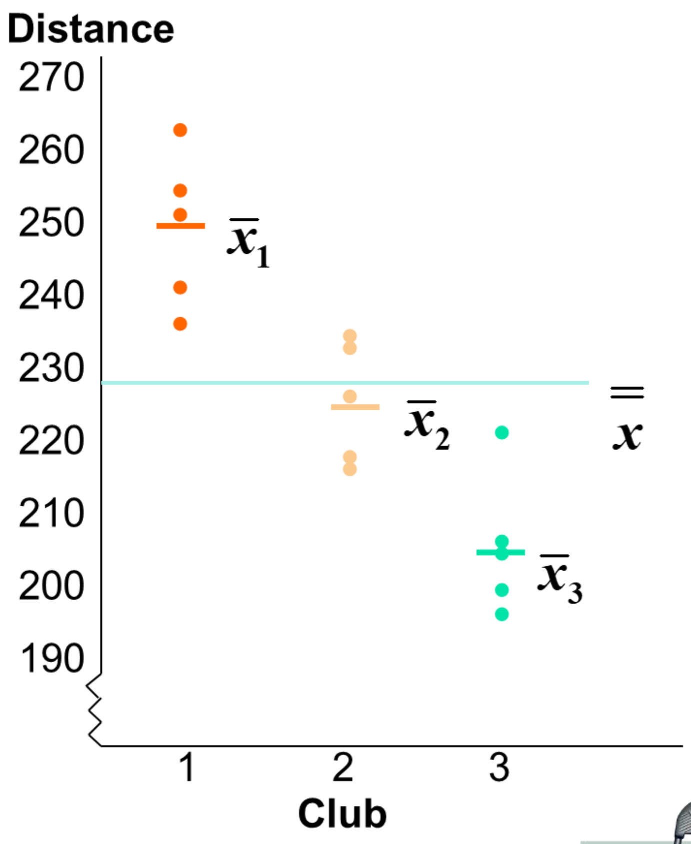
| Club 1 | Club 2 | Club 3 |
|---|---|---|
| 254 | 234 | 200 |
| 263 | 218 | 222 |
| 241 | 235 | 197 |
| 237 | 227 | 206 |
| 251 | 216 | 204 |

# One-Factor ANOVA Example: Scatter Diagram

| Club 1 | Club 2 | Club 3 |
|---|---|---|
| 254 | 234 | 200 |
| 263 | 218 | 222 |
| 241 | 235 | 197 |
| 237 | 227 | 206 |
| 251 | 216 | 204 |

$$\bar{x}_1 = 249.2 \quad \bar{x}_2 = 226.0 \quad \bar{x}_3 = 205.8$$

$$\bar{\bar{x}} = 227.0$$




# One-Factor ANOVA Example Computations

| Club 1 | Club 2 | Club 3 |
|---|---|---|
| 254 | 234 | 200 |
| 263 | 218 | 222 |
| 241 | 235 | 197 |
| 237 | 227 | 206 |
| 251 | 216 | 204 |

$$\bar{x}_1 = 249.2 \qquad n_1 = 5$$
$$\bar{x}_2 = 226.0 \qquad n_2 = 5$$
$$\bar{x}_3 = 205.8 \qquad n_3 = 5$$
$$\bar{\bar{x}} = 227.0 \qquad n = 15$$
$$k = 3$$

$$SSG = 5(249.2-227)^2 + 5(226-227)^2 + 5(205.8-227)^2 = 4716.4$$
$$SSW = (254-249.2)^2 + (263-249.2)^2 + \ldots + (204-205.8)^2 = 1119.6$$

$$MSG = \frac{4716.4}{(3-1)} = 2358.2$$
$$MSW = \frac{1119.6}{(15-3)} = 93.3$$
$$F = \frac{2358.2}{93.3} = 25.275$$

# One-Factor ANOVA Example Solution

$H_0: \mu_1 = \mu_2 = \mu_3$

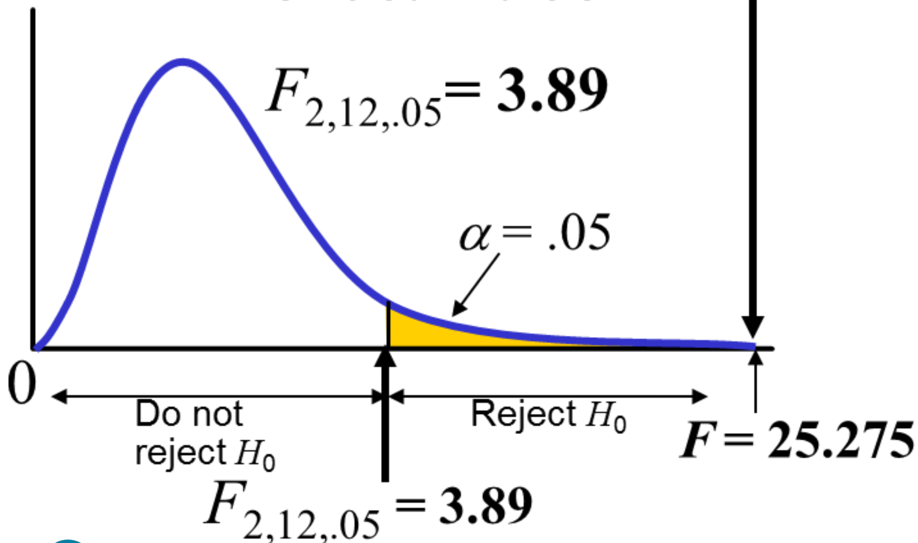
$H_1: \mu_i$ not all equal

$\alpha = .05$

$\text{df}_1 = 2 \qquad \text{df}_2 = 12$

**Critical Value:**

$F_{2,12,.05} = \mathbf{3.89}$




**Test Statistic:**

$$F = \frac{\text{MSA}}{\text{MSW}} = \frac{2358.2}{93.3} = 25.275$$

**Decision:**

Reject $H_0$ at $\alpha = 0.05$

**Conclusion:**

There is evidence that at least one $\mu_i$ differs from the rest

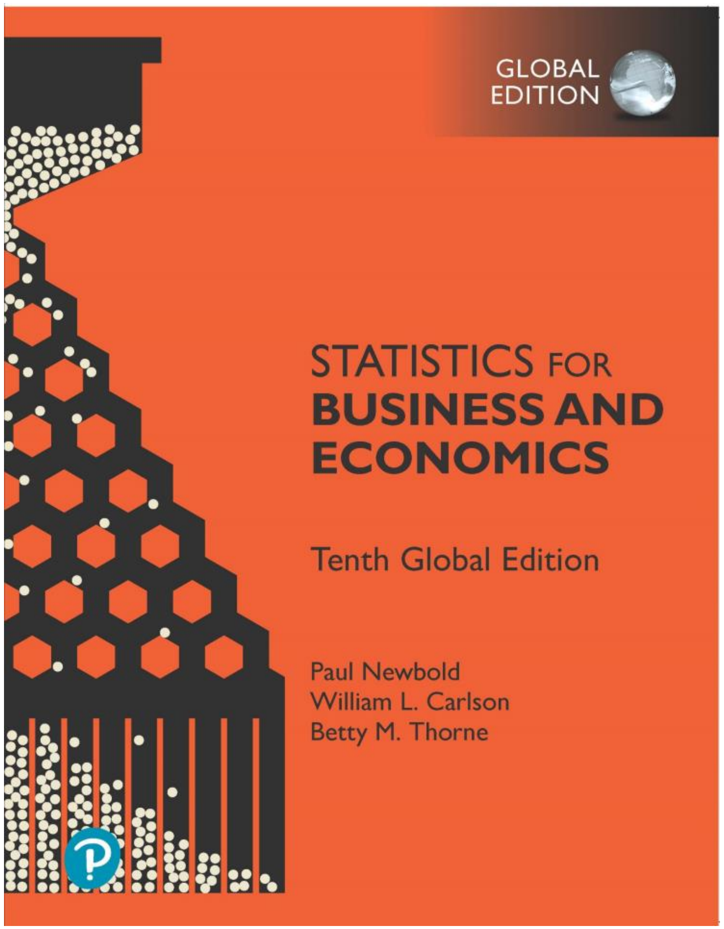
# ANOVA -- Single Factor: Excel Output

Excel: data | data analysis | ANOVA: single factor

| | | | | | | |
|---|---|---|---|---|---|---|
| **Summary** | | | | | | |
| Groups | Count | Sum | Average | Variance | | |
| Club 1 | 5 | 1246 | 249.2 | 108.2 | | |
| Club 2 | 5 | 1130 | 226 | 77.5 | | |
| Club 3 | 5 | 1029 | 205.8 | 94.2 | | |
| **ANOVA** | | | | | | |
| Source of Variation | SS | df | MS | *F* | *P*-value | *F* crit |
| Between Groups | **4716.4** | **2** | **2358.2** | **25.275** | 4.99E-05 | **3.89** |
| Within Groups | **1119.6** | **12** | **93.3** | | | |
| Total | 5836.0 | 14 | | | | |

# Statistics for Business and Economics

Tenth Edition, Global Edition




## Chapter 14

## Analysis of Categorical Data

# Chapter Goals

- Use the chi-square goodness-of-fit test to determine whether data fits specified probabilities
- Set up a contingency analysis table and perform a chi-square test of association

# Introduction

- Nonparametric Statistics
  - Fewer restrictive assumptions about data levels and underlying probability distributions
    - Population distributions may be skewed
    - The level of data measurement may only be ordinal or nominal

# Chi-Square Goodness-of-Fit Test

- Are technical support calls equal across all days of the week? (i.e., do calls follow a uniform distribution?)
  - Sample data for 10 days per day of week:

| | Sum of calls for this day: |
|---|---|
| Monday | 290 |
| Tuesday | 250 |
| Wednesday | 238 |
| Thursday | 257 |
| Friday | 265 |
| Saturday | 230 |
| Sunday | 192 |

$$\sum = 1722$$

# Logic of Goodness-of-Fit Test

- If calls **are** uniformly distributed, the 1722 calls would be expected to be equally divided across the 7 days:

$$\frac{1722}{7} = 246 \text{ expected calls per day if uniform}$$

- Chi-Square Goodness-of-Fit Test: test to see if the sample results are consistent with the expected results

# Observed vs. Expected Frequencies (1 of 4)

| | Observed $O_i$ | Expected $E_i$ |
|---|---|---|
| Monday | 290 | 246 |
| Tuesday | 250 | 246 |
| Wednesday | 238 | 246 |
| Thursday | 257 | 246 |
| Friday | 265 | 246 |
| Saturday | 230 | 246 |
| Sunday | 192 | 246 |
| Total | 1722 | 1722 |

# Chi-Square Test Statistic (1 of 2)

$H_0:$ The distribution of calls is uniform over days of the week

$H_1:$ The distribution of calls is not uniform

- The test statistic is

$$\chi^2 = \sum_{i=1}^{K} \frac{(O_i - E_i)^2}{E_i} \quad (\text{where d.f.} = K - 1)$$

where:

$K$ = number of categories

$O_i$ = observed frequency for category *i*

$E_i$ = expected frequency for category *i*

# The Rejection Region

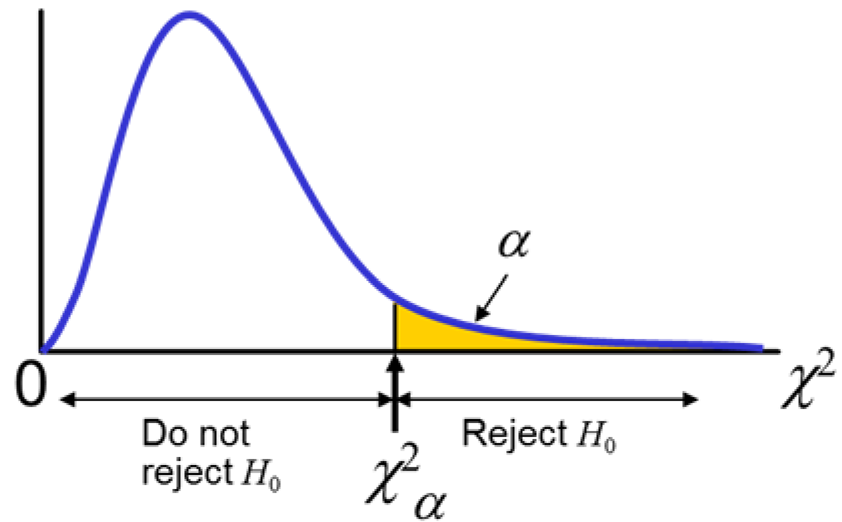
$H_0$ : The distribution of calls is uniform over days of the week

$H_1$ : The distribution of calls is not uniform

$$\chi^2 = \sum_{i=1}^{K} \frac{(O_i - E_i)^2}{E_i}$$

- Reject $H_0$ if $\chi^2 > \chi^2_\alpha$

(with $k$ − 1 degrees of freedom)



# Observed vs. Expected Frequencies (2 of 4)

| | Observed $O_i$ | Expected $E_i$ | $(O_i - E_i)$ | $(O_i - E_i)^2$ | $\frac{(O_i - E_i)^2}{E_i}$ |
|---|---|---|---|---|---|
| Monday | 290 | 246 | 44 | 1936 | 7.870 |
| Tuesday | 250 | 246 | 4 | 16 | 0.065 |
| Wednesday | 238 | 246 | −8 | 64 | 0.260 |
| Thursday | 257 | 246 | 11 | 121 | 0.492 |
| Friday | 265 | 246 | 19 | 361 | 1.467 |
| Saturday | 230 | 246 | −16 | 256 | 1.041 |
| Sunday | 192 | 246 | −54 | 2916 | 11.854 |
| Total | 1722 | 1722 | | | $\chi^2 = 23.049$ |

# Chi-Square Test Statistic (2 of 2)

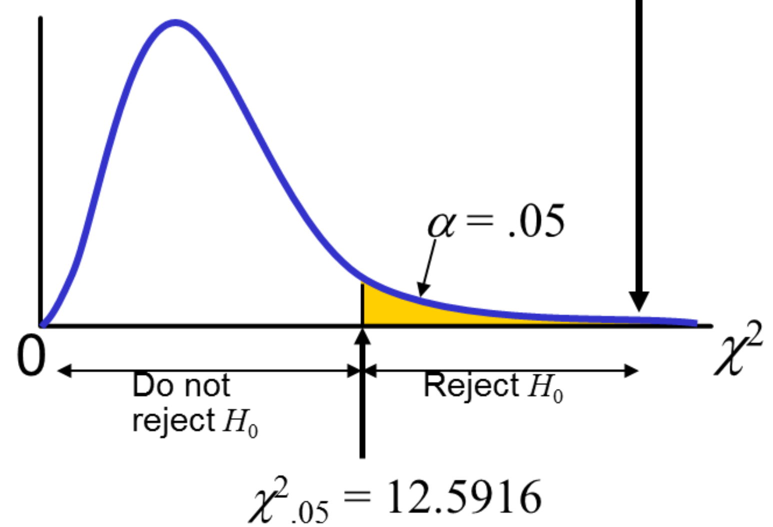
$H_0$ : The distribution of calls is uniform over days of the week

$H_1$ : The distribution of calls is not uniform

$$\chi^2 = \frac{(290-246)^2}{246} + \frac{(250-246)^2}{246} + \ldots + \frac{(192-246)^2}{246} = \boxed{23.049}$$

*K* − 1 = 6 (7 days of the week) so use 6 degrees of freedom:

$$\chi^2_{.05} = 12.5916$$




**Conclusion:**

$$\chi^2 = 23.05 > \chi^2_{\alpha} = 12.5916$$

so **reject $H_0$** and conclude that the distribution is not uniform

# Section 14.3 Contingency Tables

Contingency Tables

- Used to classify sample observations according to a pair of attributes
- Also called a cross-classification or cross-tabulation table
- Assume *r* categories for attribute A and *c* categories for attribute B
  - Then there are $(r \times c)$ possible cross-classifications

# $r$ Times $c$ Contingency Table

| | Attribute B | | | | |
|---|---|---|---|---|---|
| **Attribute A** | **1** | **2** | **. . .** | ***c*** | **Totals** |
| **1** | $O_{11}$ | $O_{12}$ | ... | $O_{1c}$ | $R_1$ |
| **2** | $O_{21}$ | $O_{22}$ | ... | $O_{2c}$ | $R_2$ |
| **.** | . | . | ... | . | . |
| **.** | . | . | ... | . | . |
| **.** | . | . | ... | . | . |
| ***r*** | $O_{r1}$ | $O_{r2}$ | ... | $O_{rc}$ | $R_r$ |
| **Totals** | $C_1$ | $C_2$ | ... | $C_c$ | $n$ |

# Test for Association (1 of 2)

- Consider *n* observations tabulated in an $r \times c$ contingency table
- Denote by $O_{ij}$ the number of observations in the cell that is in the *i*th row and the *j*th column
- The null hypothesis is

$H_0$ : No association exists between the two attributes in the population

- The appropriate test is a chi-square test with $(r-1)(c-1)$ degrees of freedom

# Test for Association (2 of 2)

- Let $R_i$ and $C_j$ be the row and column totals
- The expected number of observations in cell row *i* and column *j*, given that $H_0$ is true, is

$$E_{ij} = \frac{R_i C_j}{n}$$

- A test of association at a significance level *α* is based on the chi-square distribution and the following decision rule

Reject $H_0$ if $\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}} > \chi^2_{(r-1)(c-1),\alpha}$

# Contingency Table Example (1 of 2)

Left-Handed vs. Gender

- Dominant Hand: Left vs. Right
- Gender: Male vs. Female

$H_0:$ There is no association between hand preference and gender

$H_1:$ Hand preference is not independent of gender

# Contingency Table Example (2 of 2)

Sample results organized in a contingency table:

sample size = $n$ = 300:

120 Females, 12 were left handed

180 Males, 24 were left handed

| Gender | Hand Preference | | |
|---|---|---|---|
| | Left | Right | |
| Female | 12 | 108 | 120 |
| Male | 24 | 156 | 180 |
| | 36 | 264 | 300 |

# Logic of the Test

$H_0:$ There is no association between hand preference and gender

$H_1:$ Hand preference is not independent of gender

- If $H_0$ is true, then the proportion of left-handed females should be the same as the proportion of left-handed males
- The two proportions above should be the same as the proportion of left-handed people overall

# Finding Expected Frequencies

120 Females, 12 were left handed

180 Males, 24 were left handed

→

**Overall:**

$$P(\text{Left Handed}) = \frac{36}{300} = .12$$

**If no association, then**

$$P(\text{Left Handed} \mid \text{Female}) = P(\text{Left Handed} \mid \text{Male}) = .12$$

So we would expect 12% of the 120 females and 12% of the 180 males to be left handed…

**i.e., we would expect** $\mathbf{(120)(.12) = 14.4}$ **females to be left handed**

$\mathbf{(180)(.12) = 21.6}$ **males to be left handed**

# Expected Cell Frequencies

- Expected cell frequencies:

$$E_{ij} = \frac{R_i C_j}{n} = \frac{\left(i^{\text{th}}\ \text{Row total}\right)\left(j^{\text{th}}\ \text{Column total}\right)}{\text{Total sample size}}$$

Example:

$$E_{11} = \frac{(120)(36)}{300} = 14.4$$

# Observed vs. Expected Frequencies (3 of 4)

Observed frequencies vs. expected frequencies:

| Gender | Hand Preference | | |
|---|---|---|---|
| | Left | Right | |
| Female | Observed = 12<br>Expected = 14.4 | Observed = 108<br>Expected = 105.6 | 120 |
| Male | Observed = 24<br>Expected = 21.6 | Observed = 156<br>Expected = 158.4 | 180 |
| | 36 | 264 | 300 |

# The Chi-Square Test Statistic

The Chi-square test statistic is:

$$\chi^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}} \quad \text{with} \quad \text{d.f.} = (r-1)(c-1)$$

- where:

$O_{ij}$ = observed frequency in cell (*i*, *j*)

$E_{ij}$ = expected frequency in cell (*i*, *j*)

*r* = number of rows

*c* = number of columns

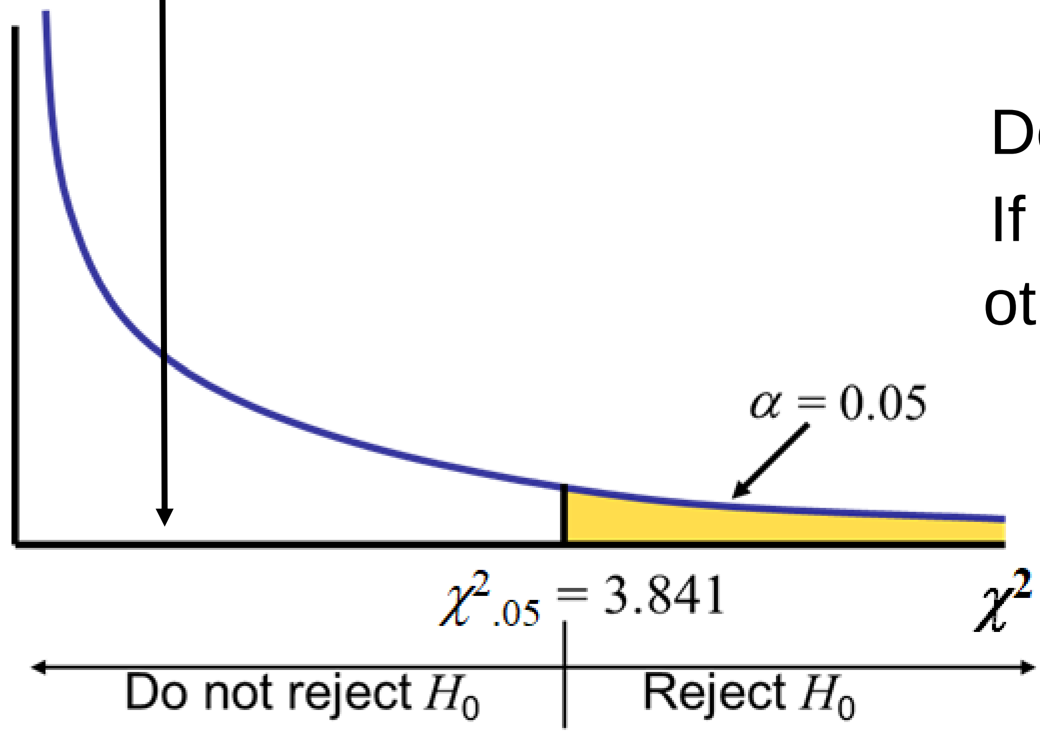
# Observed vs. Expected Frequencies (4 of 4)

| Gender | Hand Preference | | |
|---|---|---|---|
| | Left | Right | |
| Female | Observed = 12<br>Expected = 14.4 | Observed = 108<br>Expected = 105.6 | 120 |
| Male | Observed = 24<br>Expected = 21.6 | Observed = 156<br>Expected = 158.4 | 180 |
| | 36 | 264 | 300 |

$$\chi^2 = \frac{(12-14.4)^2}{14.4} + \frac{(108-105.6)^2}{105.6} + \frac{(24-21.6)^2}{21.6} + \frac{(156-158.4)^2}{158.4} = 0.7576$$

# Contingency Analysis

$\chi^2 = \boxed{0.7576}$ with $\text{d.f.} = (r-1)(c-1) = (1)(1) = 1$




Decision Rule:

If $\chi^2 > 3.841$, reject $H_0$, otherwise, do not reject $H_0$

Here, $\chi^2 = 0.7576 < 3.841$, so we do not reject $H_0$ and conclude that gender and hand preference are not associated